# Effect of ultrashort pulsed illumination on foci caused by a Fresnel zone plate

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The focal distribution produced by a zone plate under ultrashort pulsed laser illumination is investigated under the Fresnel approximation. A comparison of the diffraction patterns in the focal region between pulsed and continuous-wave illumination shows that the focal shape produced by a zone plate can be significantly altered when an ultrashort pulse is shorter than 100 fs. In particular, the focal width in the axial and the transverse directions is increased by approximately 5% and 85%, respectively, from continuous-wave illumination to 10-fs pulsed illumination. © 2003 Optical Society of America

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Recently, the femtosecond pulsed-beam technology has advanced to such a stage that it is possible to produce an infrared, ultrashort, pulsed beam of tens of femtosecond or less in pulse width.[1] Such a short pulsed beam corresponds to a broad spectrum. For example, for a 5-fs pulsed beam, the spectral width is approximately 400 nm.[2] Consequently, diffraction by a femtosecond pulsed beam may be significantly different from that by a monochromatic continuous-wave (cw) beam.[3–10] It has been found that the broad spectrum from a femtosecond pulsed beam tends to smooth the diffraction pattern by circular apertures,[4] serrated apertures,[4] circular disks,[6] and serrated disks.[6,7] The effect of the broad spectrum on diffraction patterns becomes significant when the transmission function of diffraction elements is wavelength dependent.[8–10] In the case of a lens illuminated with an ultrashort pulsed beam, the wavelength-dependent dispersion relation results in radius-dependent pulse broadening and group-velocity dispersion.[8] As a result, the focal spot by the lens is severely distorted.

Another important diffraction element for focusing is a zone plate consisting of a series of alternative obstructed and unobstructed concentric zones of appropriate radii.[11] The radii $R_m$ of the zones of a zone plate can be expressed[12,13]

$$R_m{}^2 = mf\lambda, \tag{1}$$

where $m$ is an integer, $\lambda$ is the illumination wavelength, and $f$ is the focal length one may require. However, to the best of authors' knowledge there has been no report that discusses the effect of the wavelength-dependent transmittance of a zone plate shown in Eq. (1) on its focal distribution under femtosecond pulsed illumination, except for the estimation of chromatic aberation.[11] Our aim here is to examine this effect.

A transmission zone plate for focusing a monochromatic beam of light has been studied both theoretically[12] and experimentally.[13] In addition to the principal focus at $z = f$, there are secondary foci at $f/(2n + 1)$, where $n$ is an integer. Under the circularly symmetric condition, the diffraction pattern by a zone plate at a frequency $\omega$ can be expressed, under the Fresnel approximation, as[4,14]

$$U(\rho, z, \omega) = \frac{i2N \exp(-i2\pi z/\lambda)}{\lambda z} \sum_{m=0}^{M_0} \int_{\rho_m}^{\rho_{m+1}} \times U_0(\rho_1, \omega) J_0(2N\rho\rho_1) \exp(-iN\rho_1{}^2)\rho_1 d\rho_1, \tag{2}$$

where

$$\rho_m{}^2 = \frac{R_m{}^2}{a^2} = \frac{mf\lambda}{a^2} \tag{3}$$

and $N$ is the Fresnel number defined as

$$N = \frac{\pi a^2}{\lambda z}. \tag{4}$$

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The integer number $M_0$ is the maximum of $m$. $J_0$ is the Bessel function of the first kind of order zero. $U_0$ is the field distribution over the aperture of the zone plate. Equation (2) implies that the diffraction pattern is contributed from spherical wavelets with a propagation direction close to the axis.[14] In that sense, this treatment differs from the classical method for a zone plate, in which plane wavelets are used.[12,13]

Under femtosecond pulsed illumination, let us assume that its spectral distribution is given a Gaussian profile.[4] The time-averaged diffraction intensity caused by a zone plate is thus given by[4,14]

$$I(\rho, N_0) = C \int_0^{+\infty} \frac{1}{N_0} \left| \exp\left[ -\frac{T^2 \omega_0^{\,2}}{4} \left( \frac{N}{N_0} - 1 \right)^2 \right] \times \sum_{m=0}^{M_0} \int_{\rho_m}^{\rho_{m+1}} J_0(2N\rho\rho_1) \times \exp(-iN\rho_1^{\,2}) \rho_1 \mathrm{d}\rho_1 \right|^2 \mathrm{d}N, \tag{5}$$

where $N_0$ is defined by the center wavelength $\lambda_0$ (or the center frequency $\omega_0$) within the spectral profile. $T$ corresponds to the full temporal width at half-maximum.[4]

The intensity distribution of Eqs. (2) and (5) can be numerically calculated and is shown in Fig. 1. The axial coordinate $Z$ in Fig. 1 is $1/N_0$, and the position of the principal focus is at $N_0 = 34$. As expected, there are a series of foci along the axis. The size of the focal spot decreases with the axial distance. In the case of femtosecond pulsed illumination [Figs. 1(b) and 1(c)], the size of the foci becomes broadened in particular when the pulse width is less than 10 fs.

To quantify these changes, let us consider the on-axis intensity distribution when $\rho = 0$. In this case, the diffracted intensity distribution can be derived as

$$I(N_0) = \frac{\sin^2\left( \dfrac{m_0 \pi N_0}{N_f} \right)}{\cos^2\left( \dfrac{\pi N_0}{2N_f} \right)} \tag{6}$$

for cw illumination and

$$I(N_0) = C \int_0^{+\infty} \frac{1}{N_0} \left| \exp\left[ -\frac{T^2 \omega_0^{\,2}}{4} \left( \frac{N}{N_0} - 1 \right)^2 \right] \times \left[ \frac{\sin\left( \dfrac{m_0 \pi N}{N_f} \right)}{\cos\left( \dfrac{\pi N}{2N_f} \right)} \right] \right|^2 \mathrm{d}N \tag{7}$$

for femtosecond pulsed illumination. Here $N_f$ is the Fresnel number corresponding to $z = f$ at the center wavelength and $m_0$ is the number of the unobstructed zones.

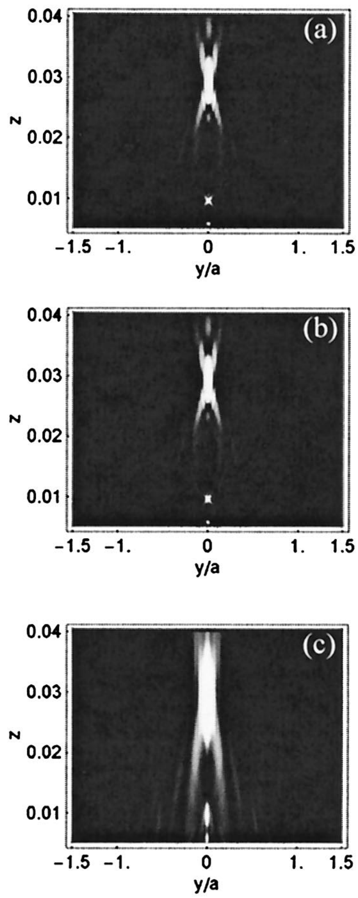


Fig. 1. Normalized intensity distributions in the region of the principal focus along the axial plane. The plot range corresponds to the variation of the Fresnel number $N_0$ from 200 to 25 and $m_0 = 6$. Part (a), cw illumination; (b), 100-fs pulsed illumination; (c), 10-fs pulsed illumination.

Equations (6) and (7) show that the focal positions are located at $z = f, f/3, f/5$, as expected. The intensity at the foci for cw illumination is equal [Fig. 2(a)], which differs from the prediction from the classical theory[12] because spherical wavelets have been considered in Eq. (2). Note that the Fresnel approximation in Eq. (2) does not hold if the observation point is close to the diffraction screen.[4,15] In this case, the generalized Fresnel diffraction theory may lead to an unequal intensity at foci. On the other hand, however, the intensity at foci under femtosecond pulsed illumination may be significantly changed. This feature is confirmed in Figs. 2(b) and 2(c); the focal intensity reduces as the axial distance decreases. In addition, the axial width of the focal spots is significantly increased, compared with Fig. 2(a). These changes become noticeable when the pulse width is 100 fs. For 10-fs pulsed illumination, the intensity of the third focus is only 10% of the principal focus, and the full width at half-maximum (FWHM) of the principal

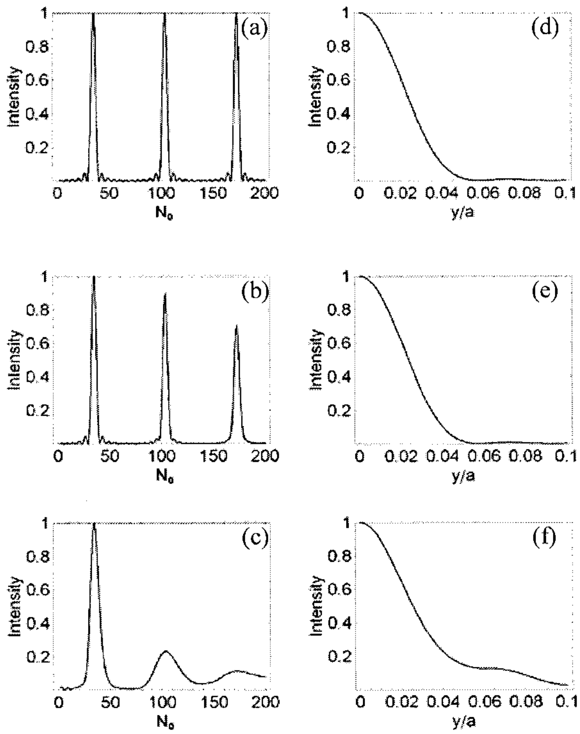


Fig. 2. Axial [(a), (b), (c)] and transverse [(d), (e), (f)] normalized intensity profiles. The transverse intensity profiles are given at $N_0 = 34$ and $m_0 = 6$. Parts (a) and (d) are under cw illumination; (b) and (e) are under 100-fs pulsed illumination; (c) and (f) are under 10-fs pulsed illumination.

focus in Fig. 2(c) is increased by approximately 85% compared with that in Fig. 2(a).

The transverse size of the principal focal spot is shown in Figs. 2(d)–2(f). It is evident that the broadening in the transverse direction, caused by femtosecond pulsed illumination, is not as large as that in the axial direction. For example, the transverse FWHM in Fig. 2(f) is only 5% larger than that in Fig. 2(e). However, the zero-intensity minimum under cw illumination in Fig. 2(d) disappears when the pulse width is 10 fs.

The physical reason for these dramatic changes described in Figs. (1) and (2) is the broad spectrum of a femtosecond pulse beam. Each wavelength components produces a diffraction pattern with maximum and minimum positions determined by the given wavelength. The superposition of these patterns leads to the broadening of the focal spots. In Fig. 3, the dependence of the FWHM of the principal focus in the axial and the transverse directions on the pulse width is exhibited, together with the Fresnel number width $\Delta N$ corresponding to the spectral width. It is clear that the effect of a femtosecond pulsed beam becomes noticeable when the pulsed width is shorter than 30 fs and becomes severe if the pulse width is less than 10 fs, which corresponds to the fractional bandwidth of 0.238 at a wavelength of 800 nm for a Gaussian-shaped pulsed beam.[14]

In conclusion, the effect of femtosecond pulsed il-

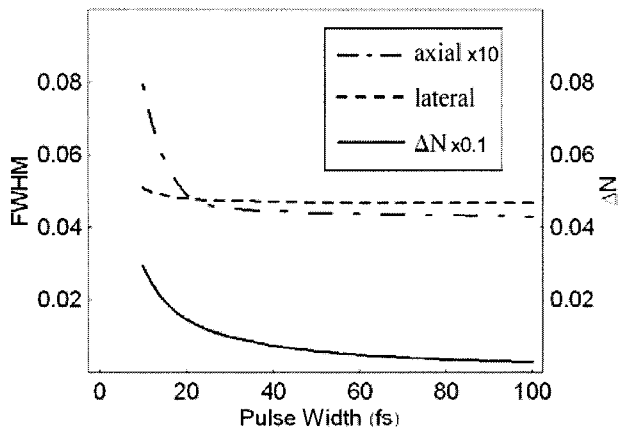


Fig. 3. Dependence of the axial and the lateral focal widths, $\Delta Z$ and $\Delta(y/a)$ on the pulse width. The values of $\Delta Z$ and $\Delta N$ have been multiplied by a factor of 10 and 0.1, respectively.

lumination on the focal intensity distribution caused by a zone plate has been numerically studied. The focal-spot size can be significantly broadened by the broad spectrum of a femtosecond pulsed beam. This conclusion means that a zone plate may not suitable for focusing a femtosecond pulsed beam if there is no compensation for the distortion. However, such compensation can be easily introduced, because the chromatic aberration caused by a zone plate is radial dependent only.[11]

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